The Interface between Quantum Mechanics and General Relativity

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Lamb Medal Lecture of January 5, 2006
(Writeup for the Proceedings of the 36th Winter Colloquium on the Physics of Quantum Electronics at Snowbird, Utah (version of June 16, 2006 for George Welch and for the ArXiv))

Abstract

The generation, as well as the detection, of gravitational radiation by means of charged superfluids is considered. One example of such a "charged superfluid" consists of a pair of Planck-mass-scale, ultracold "Millikan oil drops," each with a single electron on its surface, in which the oil of the drop is replaced by superfluid helium. When levitated in a magnetic trap, and subjected to microwave-frequency electromagnetic radiation, a pair of such "Millikan oil drops" separated by a microwave wavelength can become an efficient quantum transducer between quadrupolar electromagnetic and gravitational radiation. This leads to the possibility of a Hertz-like experiment, in which the source of microwave-frequency gravitational radiation consists of one pair of "Millikan oil drops" driven by microwaves, and the receiver of such radiation consists of another pair of "Millikan oil drops" in the far field driven by the gravitational radiation generated by the first pair. The second pair then back-converts the gravitational radiation into detectable microwaves. The enormous enhancement of the conversion efficiency for these quantum transducers over that for electrons arises from the fact that there exists macroscopic quantum phase coherence in these charged superfluid systems.

The equivalence principle revisited: Does a falling charge radiate?

Galileo first performed experiments demonstrating that all freely-falling objects, independent of their mass, accelerate downwards with the same acceleration **g** due to Earth's gravity. Later, Eötvös, and still later, Dicke, performed more sensitive experiments, which showed that this statement of the equivalence prin-

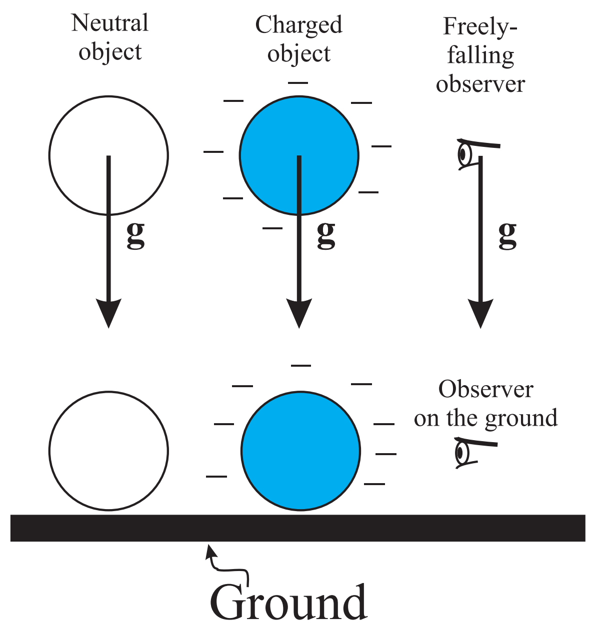

Figure 1: Equivalence principle for a neutral and a charged object.

ciple was true to extremely high accuracy, independent of the mass and of the composition of these objects [1].

One might therefore expect that a neutral object and a charged object, when simultaneously dropped from the same height, would hit the ground at the same instant. See Figure 1.

However, a well-known paradox [2] now arises when we ask the following question: Is it the falling charged object, or is it the stationary charged object at rest on the ground, that radiates electromagnetic waves?

On the one hand, a freely-falling observer, who is co-moving with the freely falling neutral and charged objects, sees these two objects as if they were freely floating in space. The falling charged object would therefore appear to him not to be accelerating, so that he would conclude that it is not this charge which radiates. Rather, when he looks downwards at the charged object which is at rest on the ground, he sees a charge which is accelerating upwards with an acceleration $-\mathbf{g}$ towards him. He would therefore conclude that it is the charged object at rest on the ground, and not the falling charge, that is radiating electromagnetic radiation.

On the other hand, an observer on the ground would come to the opposite conclusion. She sees the falling charge accelerating downwards with an acceleration $\mathbf{g}$ towards her, whereas the charged object at rest on the ground does not appear to her to be undergoing any acceleration. She would therefore conclude that it is the falling charge which radiates electromagnetic radiation, and not

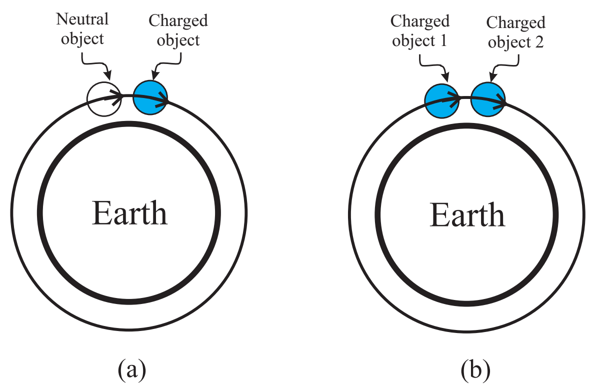

Figure 2: (a) Circular orbit around the Earth of a neutral object and a charged object. (b) Circular orbit of two charged objects.

the charge which is resting on the ground. Which conclusion is the correct one?

As a first step towards the resolution of this paradox, we note that the concept of "radiation" makes sense only in the far field of moving charged sources *asymptotically*. We must therefore ask the further question: What would an observer at infinity see?

Motivated by this further question, let us change the setting for the formulation of this paradox to that of two nearby objects, one neutral and one charged, orbiting in free fall around the Earth in the same circular orbit, as seen by a distant observer. See Figure 2(a).

It now becomes clear that the charged object will gradually spiral in towards the Earth, since it is undergoing constant centripetal acceleration in uniform circular motion, and will therefore in principle lose energy due to the emission of electromagnetic radiation at a rate determined by Larmor's radiation-power formula

$$P_{EM} = \frac{2}{3}\frac{q^2}{c^3}a^2 \tag{1}$$

where P_{EM} is the total amount of power emitted in electromagnetic radiation by the charge q undergoing centripetal acceleration a. The energy escaping to infinity in the form of electromagnetic radiation emitted by the orbiting charged object must come from its gravitational potential energy (which is related by the virial theorem to its kinetic energy), and therefore this object will gradually spiral inwards towards the surface of the Earth. This kind of decaying orbital motion is the same as that of the electron in Bohr's planetary model of the hydrogen atom, when the electron's motion around the proton is considered using only classical concepts. Here the classical description is clearly a valid one.

Now it is true that the neutral object will also in principle undergo orbital decay, i.e., it will also gradually spiral inwards towards the Earth's surface, due

to the gradual loss of energy arising from the emission of gravitational radiation in accordance with the gravitational form of Larmor's radiation-power formula

$$P'_{GR} = \kappa \frac{2}{3} \frac{Gm^2}{c^3} a^2 \tag{2}$$

where κ is a numerical factor that accounts for the quadrupolar nature of gravitational radiation, G is Newton's constant, and m is the mass of the neutral orbiting object, which is undergoing essentially the same centripetal acceleration a as the charged object [3]. (The prime on P'_{GR} denotes the incorporation of the factor of κ into the formula for radiation power.) The decay of orbital motion due to the emission of gravitational radiation has been observed in the case of Taylor's binary pulsar PSR 1913+16 [4].

The rate of orbital decay due to the emission of gravitational radiation will be much smaller than that due to the emission of electromagnetic radiation, whenever the dimensionless ratio of coupling constants obeys the inequality

$$\frac{\kappa Gm^2}{q^2} << 1. \tag{3}$$

In such cases, one can neglect the orbital decay due to gravitational radiation as compared to that due to electromagnetic radiation.

In any case, however, the orbital motion of a charged object will always decay faster than that of a neutral object with the same mass. An astronaut would therefore see a differential motion between these two nearby objects. Even when inside a windowless spacecraft, the astronaut would still be able to tell the direction of the center of the Earth, by carefully observing the motion of the charged object relative to the neutral object inside the spacecraft, since the charged object would be gradually drifting radially towards the center of the Earth faster than the neutral object. Although this effect may be extremely small, and may be masked by large systematic errors, we are discussing here matters of principle. Here the principle of the conservation of energy demands the existence of this kind of differential motion.

Is the equivalence principle violated?

The above prediction of a differential motion between charged and neutral objects in Earth's orbit seems at first sight to violate the equivalence principle, and thus would seem to render invalid the concept of "geodesic" in general relativity, which demands that all freely-falling material objects, independent of their mass and composition (including charge), traverse the same shortest (geodesic) path in spacetime connecting any two given spacetime points.

However, it must be kept in mind that the equivalence principle implicitly assumes that any such material object is to be viewed as a "vanishingly small" test mass, and furthermore implicitly assumes that any charge associated with this test mass is be to viewed also as a test charge, whose charge is also "vanishingly small." One is employing here the usual limiting procedure involving

test particles to define the local value of a classical field, both gravitational and electrical [5].

By contrast, a *finitely* charged object experiences a nonvanishing electromagnetic force due to radiation damping, which is an effectively viscous kind of force. This implies that a finitely charged object is undergoing approximately, but not truly exactly, *free* fall. Hence there is no reason to believe that a finitely charged object would follow a neutral object along the same geodesic, and the equivalence principle is therefore not violated.

Charge, at a fundamental level, is a quantum concept. Dirac's charge-monopole quantization rule shows that the quantization of electrical charge arises from global, topological, and quantum-mechanical considerations. The fact that charge is quantized in integer values of the electron charge e, stands in contradiction with the usual limiting procedure that is used in all classical field theories to define the concept of "field," in which it is assumed that the test charge (or test mass) which is used to measure the local value of the field, is a continuous variable that can be smoothly reduced to zero.

In this classical procedure of taking the test-particle limit, one can neglect the quantum "back-action" of the test particle back onto the field, because the charge and the mass both smoothly go to zero, and therefore any back-actions that the test particle might have caused onto the classical electromagnetic and gravitational fields, must also go smoothly to zero. However, for a particle with a finite, quantized charge and mass, for example, for a single electron, this "no quantum back-action" assumption violates the uncertainty principle.

Therefore quantized charged systems are a good place to examine the conceptual tensions that lie at the interface of quantum mechanics and general relativity [6]. As will be argued below, single-electron–charged, macroscopically phase-coherent quantum fluids are particularly promising systems in which to discover experimentally new phenomena that might emerge from these conceptual tensions.

Two charged objects orbiting the Earth

Now let us examine the details of the motion of two finitely charged objects orbiting around the Earth. See Figure 2(b).

For concreteness, imagine that these two charged objects are two Millikan oil drops with single electrons attached to them, which are nearby to each other in the same circular orbit. How massive would these oil drops have to be before the mutual repulsion due to the electrical force between them, changes to a mutual attraction due to the gravitational force? When they exceed a certain critical mass, one expects that the drops will drift towards each other, rather than drifting farther apart. We shall calculate this critical mass presently.

Now imagine what would happen if a low-frequency gravity wave passes over these two Millikan oil drops, when this wave propagates at normal incidence into the plane of the orbit. Such a wave would exert a time-varying tidal gravitational force, which would alternately stretch and squeeze sinusoidally in time

the space between these objects, when one of the polarization axes of the gravity wave is chosen to be aligned with respect to the line connecting the two drops. Therefore the distance between these charged objects would become an oscillating function of time, according to the observer at infinity, and this implies the emission of electromagnetic radiation by these approximately freely-falling objects. Thus this two-Millikan-oil-drop system would be a kind of transducer, in which gravitational radiation can be converted into electromagnetic radiation in a scattering process. For weak radiation fields, such a conversion process would be linear and reciprocal in nature.

However, for very high-frequency gravity waves, it would be possible to excite a very large number of internal degrees of freedom of the classical liquid inside a given Millikan oil drop, so that the branching ratio for the conversion of gravitational wave energy into the electromagnetic wave channel, as compared to the very large number of possible internal sound and heat channels, would be extremely small, just as is the case for the classical Weber bar. For in the reciprocal process, when one attempts to use a Weber bar as a generator of gravity waves using its fundamental acoustical mode, the branching ratio for the generation of gravitational radiation power relative to that of heat generation, has been calculated to be vanishingly small [7].

The solution to the problem of the extremely small detection efficiency of gravitational radiation antennas composed of classical matter, as we shall argue below, is to freeze out all the internal acoustical and thermal degrees of freedom of matter at very low temperatures [8], and to replace the classical matter by macroscopically coherent quantum matter. For example, instead of the Weber bar, one could use a pair of well separated, ultracold, levitated singly-charged superfluid helium drops, where only their center-of-mass degrees of freedom can be excited. There results a zero-phonon, Mössbauer-like motion of an entire superfluid drop relative to the other drop in response to the application of high-frequency gravitational or electromagnetic radiation, which can efficiently generate, as well as detect, gravitational radiation.

In the original Mössbauer effect, an excited nucleus of a certain isotope doped into a crystal can emit a gamma ray, without the usually large Doppler shift that accompanies the recoil of the emitting nucleus in the vacuum, because this nucleus is now tightly bound to the lattice. Since the vibrations of the lattice are quantized into an integer number of phonons, it is impossible for the system to emit a fraction of a quantum of sound. There results a large probability that the excited nucleus will emit the gamma ray in a zero-phonon mode. By the conservation of momentum, the recoil momentum due to the emission of the radiation must now be taken up by the center of mass of the entire system. Thus the mass of the recoiling system is the mass of the entire crystal.

This reduces the recoil Doppler shift by an enormous factor, which is on the order of the Avogadro's number of atoms present in the entire crystal. The same enormous factor also reduces the recoil Doppler shift during the absorption of the gamma ray by an unexcited nucleus of the same isotope, when this nucleus is also tightly bound to the same lattice. Extremely narrow gamma-ray resonance-fluorescence lines have therefore been observed using the same nuclear isotope

doped into two separate crystals as emitter and absorber, one crystal serving as the source, and the other as the receiver, of the radiation [9].

We shall argue below that a similar Mössbauer-like process can occur in drops of superfluid helium coated with single electrons, when they are trapped in a strong magnetic field.

Forces of gravity and electricity between two electrons

Before going on to the harder problem of electron attachment to superfluid helium drops, let us first consider the simpler problem of the forces experienced by two electrons separated by a distance r in the vacuum. Both the gravitational and the electrical force obey long-range, inverse-square laws. Newton's law of gravitation states that

$$|F_G| = \frac{Gm_e^2}{r^2} \tag{4}$$

where G is Newton's constant and m_e is the mass of the electron. Coulomb's law states that

$$|F_e| = \frac{e^2}{r^2} \tag{5}$$

where e is the charge of the electron. The electrical force is repulsive, and the gravitational one attactive.

Taking the ratio of these two forces, one obtains the dimensionless constant

$$\frac{|F_G|}{|F_e|} = \frac{Gm_e^2}{e^2} \approx 2.4 \times 10^{-43} \ . \tag{6}$$

The gravitational force is extremely small compared to the electrical force, and is therefore usually omitted in all treatments of quantum physics.

Gravitational and electromagnetic radiation powers emitted by two electrons

The above ratio of the coupling constants Gm_e^2/e^2 also is the ratio of the powers of gravitational to electromagnetic radiation emitted by two electrons separated by a distance r in the vacuum, when they undergo an acceleration a relative to each other. Larmor's formula for the power emitted by a single electron undergoing acceleration a is

$$P_{EM} = \frac{2}{3}\frac{e^2}{c^3}a^2 \ . \tag{7}$$

For the case of two electrons undergoing an acceleration a relative to each other, the radiation is quadrupolar in nature, and the modified Larmor formula (denoted by the prime) is

$$P'_{EM} = \kappa \frac{2}{3} \frac{e^2}{c^3} a^2 \ , \tag{8}$$

where the prefactor κ accounts for the quadrupolar nature of the emitted radiation. Since the electron carries mass, as well as charge, and the charge and mass co-move rigidly together, two electrons undergoing an acceleration a relative to each other will also emit quadrupolar gravitational radiation according to the formula [3]

$$P'_{GR} = \kappa \frac{2}{3} \frac{G m_e^2}{c^3} a^2 \ . \tag{9}$$

It follows that the ratio of gravitational to electromagnetic radiation powers emitted by the two-electron system is also given by

$$\frac{P'_{GR}}{P'_{EM}} = \frac{G m_e^2}{e^2} \approx 2.4 \times 10^{-43} \tag{10}$$

which involves the same ratio of coupling constants as for the ratio of the forces of gravity to electricity given by Equation (6). Thus it would seem at first sight to be hopeless to try and use the two-electron system as the means for coupling between electromagnetic and gravitational radiation.

Mössbauer-like response of electron-vortex composites

However, now consider what would happen if one were to firmly attach an electron to a vortex at the center of a small circular puddle of a nanoscale-thick thin film of superfluid helium (i.e., ^{4}He) adsorbed onto a cold substrate, which the superfluid does not wet. See Figure 3(a).

Due to the Pauli exclusion principle, the electron forms a nanoscale bubble inside superfluid helium, which is attracted to the center of the vortex by the Bernoulli effect. It then forms a bound state with the vortex with the relatively large binding energy of around 40 K or 3 meV [10]. In this local minimum-energy configuration, a tightly bound electron-vortex composite forms at the center of a circular puddle of superfluid, which possesses a circular boundary since the superfluid does not wet the substrate. Note the circular symmetry of this system.

Now imagine what would happen if the electron-vortex system were to absorb a microwave photon. See Figure 3(b).

In the zero-phonon mode of response, in which no sound waves (nor any other *quantized* deformations of the puddle at ultracold temperatures) can be emitted during the photon absorption process, a given helium atom on the edge of the puddle cannot cross the circular streamline nearest to the edge. As a

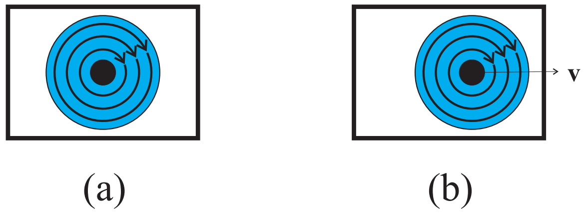

Figure 3: (a) An electron (black dot) is tightly bound to a vortex, and forms an electron-vortex composite system at the center of a circular puddle of a superfluid helium thin film adsorbed onto a cold, nonwetting substrate (rectangle). (b) When this system absorbs a microwave photon, the entire circular puddle recoils in a Mössbauer fashion.

result, this atom is constrained to follow the motion of the vortex center, along with all the other atoms which make up the entire puddle, in a Mössbauer fashion.

The circular streamlines centered on the electron at the vortex center obey the quantized-circulation condition given by the Feynman-Onsager rule [10]

$$\oint_C \mathbf{v} \cdot d\mathbf{l} = \pm 2\pi \frac{\hbar}{m} \tag{11}$$

where $\mathbf{v}$ is the velocity of the streamline in the vicinity of a differential line element $d\mathbf{l}$ of a closed curve C, $\hbar$ is Planck's constant, and m is the mass of the helium atom. The physical meaning of this quantization condition is that there is constructive interference of each helium atom with itself after one round trip around the vortex center, such as in any circular path within this kind of matter-wave, ring-interferometer configuration. The round-trip interference of the helium atom with itself is similar to that of the photon which occurs in a ring-laser–gyro configuration.

As a result of being in the zero-phonon mode, the entire electron-vortex system must recoil as a whole unit in a Mössbauer-like response to external radiation, whenever the system stays adiabatically in its zero-phonon state, which requires the use of ultralow temperatures [9]. Thus the mass of the responding system is the mass of the entire puddle.

Note that the Feynman-Onsager quantization rule is a consequence of the single-valuedness of the macroscopic wavefunction, i.e., a global quantum condition that the phase of the macroscopic wavefunction (or "complex order parameter") of the system can only change after one round trip by the quantized values of $0, \pm 2\pi, \pm 4\pi, \ldots$ Furthermore, a vortex is a topological quantum object with a hole at its center, which possesses a nonzero winding number of ± 1 corresponding to counterclockwise and clockwise senses of the superflow around the center, respectively. Moreover, the circulating currents around the vortex center can never stop flowing, i.e., there exist persistent currents of helium atoms flowing around the electron trapped at the center of the vortex, that never decay with

time. This is the behavior of a zero-loss, nonviscous charged quantum fluid.

What's the difference between quantum and classical fluids?

In light of the above, there are four answers to this question.

(1) A quantum fluid has a "quantum rigidity" due to the single-valuedness of the macroscopic wavefunction, which is absent in classical fluids. London called this property "the rigidity of the wavefunction" in the context of superconductivity, and Laughlin called this property in the context of the quantum Hall effect "an incompressible quantum fluid." This kind of rigidity arises because of the quantum adiabatic theorem, which states that when a quantum many-body system is in its ground state, it will remain adiabatically in this state in the presence of weak, slowly varying perturbations, such as those due to weak gravitational or electromagnetic radiation, provided that there is an energy gap, such as the BCS gap, or the roton gap, or the cyclotron-resonance gap, that separates the ground state from all possible excited states of the system, so that no transitions can occur to higher-energy states. Since the search for highly efficient detectors of gravitational radiation is the search for extremely rigid matter [6], quantum fluids operating in the Mössbauer mode are good candidates for high-efficiency gravity-wave antennas.

(2) A quantum fluid has a "quantum dissipationlessness." The existence of persistent currents, such as those in the electron-vortex system, is evidence for this viscosity-free, zero-loss property of a quantum fluid. Hence the generation of heat in the classical materials used in gravity wave detectors such as the Weber bar, where heat is an undesirable channel of dissipation of gravitational wave energy, is automatically closed for such quantum fluids. Thus in addition to the property of "quantum rigidity," the dissipation-free nature of quantum fluids would allow heat-free motions of superfluid helium drops, for example, in response to gravitational radiation. This frictionless property of superfluids would also greatly enhance the conversion efficiency of gravity-wave detectors based on such fluids, as compared to the extremely low efficiencies of the highly dissipative Weber bar [6].

(3) The recoil momentum upon the emission or absorption of a microwave photon by the electron-vortex composite system is taken up by the center of mass of the whole system in a Mössbauer-like effect, which is absent in a classical fluid. This is yet another aspect of the "quantum rigidity" of the quantum fluid, which does not occur classically.

(4) The entangled state of the electron-vortex system and an emitted microwave photon generated in the time-reversed version of the microwave-photon absorption process, would form a bipartite, nonlocal quantum superposition state which violates Bell's inequalities. Moreover, due to the interactions among the helium atoms, the quantum many-body system of the superfluid is automatically in a macroscopically (i.e., massively) entangled state. The quantum phase

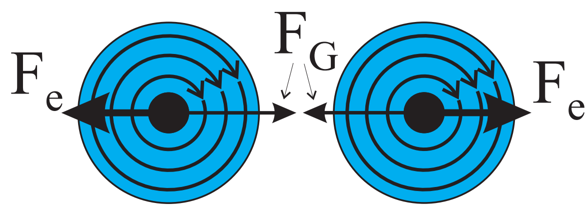

Figure 4: Comparison of the attractive gravitational force F_G with the repulsive electrical force F_e between two well-separated electron-vortex composites.

coherence of such a macroscopic superposition state would be quickly destroyed by decoherence in a classical fluid. However, here decoherence in the superfluid is prevented by the presence of an energy gap, or more generally, by the presence of a "scarcity of low-lying states," in these ultracold, macroscopically phase-coherent quantum many-body systems (i.e., "bosonic quantum fields"), in what has been called "gap-protected entanglement" [6][11].

The Planck mass scale

Let us return to the problem of the ratio of the forces of gravity and electricity, but now in the context of two well-separated electron-vortex composites at a distance r from each other. See Figure 4.

Suppose that each circular puddle contains a Planck-mass amount of superfluid helium, viz.,

$$m_{\text{Planck}} = \sqrt{\frac{\hbar c}{G}} \approx 22 \text{ micrograms} \tag{12}$$

where $\hbar$ is Planck's constant, c is the speed of light, and G is Newton's constant. Planck's mass sets the characteristic scale at which quantum mechanics ($\hbar$) impacts relativistic gravity (c, G). Note that this mass scale is *mesoscopic* [12], and not astronomical, in size. This suggests that it may be possible to perform some novel *nonastronomical*, table-top-scale experiments at the interface of quantum mechanics and general relativity, which are accessible to the laboratory.

The ratio of the forces of gravity and electricity between the two electron-vortex composites now becomes

$$\frac{|F_G|}{|F_e|} = \frac{Gm_{\text{Planck}}^2}{e^2} = \frac{G\,(\hbar c/G)}{e^2} = \frac{\hbar c}{e^2} \approx 137 \tag{13}$$

which is 45 orders of magnitude larger than the ratio given earlier by Equation (6) for the case of two electrons in the vacuum. Now the force of gravity is 137 times stronger than the force of electricity, so that instead of a mutual repulsion between these two charged objects, there is now a mutual attraction between

them. The sign change from mutual repulsion to mutual attraction between these two electron-vortex composites occurs at a critical mass m_{crit} given by

$$m_{\text{crit}} = \sqrt{\frac{e^2}{\hbar c}} m_{\text{Planck}} \approx 1.9 \text{ micrograms} \tag{14}$$

whereupon $|F_G| = |F_e|$, and the forces of gravity and electricity balance each other. This is a strong hint that mesoscopic-scale quantum effects can lead to nonnegligible couplings between gravity and electromagnetism.

The critical mass m_{crit} is also the mass at which there occurs a comparable amount of generation of electromagnetic and gravitational radiation power upon scattering of radiation from the pair of electron-vortex composites (or "Millikan oil drops," as we shall see below), each member of the pair with a mass m_{crit} and a single electron e attached to it. The ratio of quadrupolar gravitational to the quadrupolar electromagnetic radiation power ratio is given by

$$\frac{P'_{GR}}{P'_{EM}} = \frac{G m_{\text{crit}}^2}{e^2} = 1 \ , \tag{15}$$

where the numerical factors of κ in Equations (1) and (2) cancel out, since the charge of the drop co-moves together with its mass. This implies that the scattered power from these two charged objects in the gravitational wave channel becomes comparable to that in the electromagnetic wave channel. However, it should be emphasized that here we are assuming that the system's charge and mass co-move rigidly together as a single unit, in accordance with the Mössbauer-like mode of response to radiation fields. This is purely quantum effect based on the quantum adiabatic theorem's prediction that the system will remain adiabatically in its nondegenerate ground state.

Simplification to "Millikan oil drops"

From now on, we shall use the term "Millikan oil drop" with quotation marks (or, drop, without quotation marks), as the abbreviated nomenclature for "Planck-mass-scale superfluid-helium drop with a single electron firmly attached to its surface, which exhibits a Mössbauer-like response to the application of high-frequency radiation fields." By going from the 2D thin superfluid-helium film geometry of the electron-vortex composite to that a 3D superfluid-helium drop, we avoid experimental complications arising from the choice of wetting versus non-wetting substrates, and all other such substrate-related physics.

Although for simplicity we shall first consider "Millikan oil drops" with only a single electron attached to each drop, there is no reason not to consider the case also where many electrons are attached to each drop, and where a quantum Hall fluid forms on the surface of the drop in the presence of a strong magnetic field, as long as the charge-to-mass ratio of the drop is kept fixed so that the condition

$$\frac{P'_{GR}}{P'_{EM}} = 1 \tag{16}$$

is still satisfied. The quantum many-body system of the many-electron drop at ultra-low temperatures will go into its ground state, and can still possess a macroscopic amount of gravitational mass.

The helium atom is diamagnetic, and liquid helium drops have successfully been magnetically levitated in an anti-Helmholtz magnetic trapping configuration [13]. Due to its surface tension, the surface of a freely suspended, ultracold superfluid drop is atomically perfect. When an electron is approaches a drop, the formation of an image charge inside the dielectric sphere of the drop causes the electron to be attracted by the Coulomb force to its own image. However, the Pauli exclusion principle prevents the electron from entering the drop. As a result, the electron is bound to the surface of the drop in a hydrogenic ground state. Experimentally, the binding energy of the electron to the surface of liquid helium has been measured using millimeter-wave spectroscopy to be 8 Kelvin [14], which is quite large compared to the milli-Kelvin temperature scales for the proposed experiment. Hence the electron is tightly bound to the surface of the drop.

Such a "Millikan oil drop" is just as much a macroscopically phase-coherent quantum object as is the electron-vortex composite discussed earlier. In its ground state, which possesses a single, coherent quantum mechanical phase throughout the interior of the superfluid, the drop possesses a zero circulation quantum number (i.e., contains no quantum vortices), with one unit (or an integer multiple) of the charge quantum number. As a result of the drop being at ultra-low temperatures, all degrees of freedom other than the center-of-mass degrees of freedom are frozen out, so that there results a zero-phonon Mössbauer-like effect, in which the entire mass of the drop moves rigidly as a single unit in response to radiation fields. Also, since it remains adiabatically in the ground state during weak, but possibly arbitrary, perturbations due to these radiation fields, the "Millikan oil drop," like the electron-vortex composite, possesses a quantum rigidity and a quantum dissipationlessness that are the two most important quantum properties for achieving a high conversion efficiency for gravity-wave antennas.

Note that a pair of spatially separated "Millikan oil drops" have the correct quadrupolar symmetry in order to couple to gravitational radiation, as well as to quadrupolar electromagnetic radiation. When they are separated by a distance on the order of a wavelength, they become an efficient quadrupolar antenna for generating, as well as detecting, gravitational radiation.

A pair of "Millikan oil drops" as a transducer

Let us now place a pair of "Millikan oil drops" separated by approximately a microwave wavelength inside a black box, which represents a quantum transducer that can convert gravitational (GR) waves into electromagnetic (EM) waves, as indicated schematically in Figure 5(a). This kind of transducer action is similar to that discussed earlier for a low-frequency gravity wave passing over a pair of charged, freely falling objects orbiting the Earth indicated in Figure 2(b).

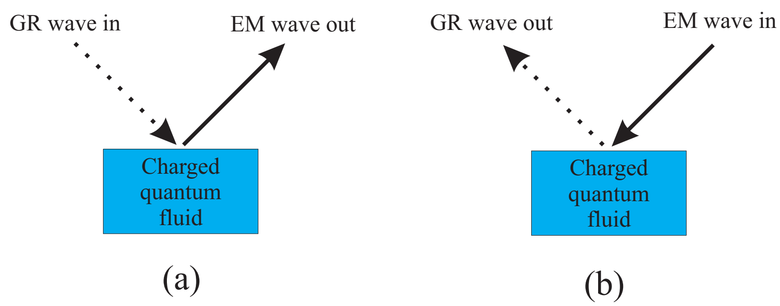

Figure 5: (a) A charged quantum fluid acts as a transducer that converts gravity waves into electromagnetic waves. (b) The reciprocal transducer action that converts electromagnetic waves into gravity waves.

It should again be stressed that these finitely charged, and approximately, but not truly exactly, freely falling objects in fact do radiate electromagnetic waves, since these waves are observable by an observer at infinity, and that the emission of this radiation does not lead to a violation of the equivalence principle, as was discussed earlier.

By time-reversal symmetry, the reciprocal process (b), as indicated in Figure 5(b), in which a charged quantum fluid such as another pair of "Millikan oil drops," converts an electromagnetic wave into a gravitational wave, must also occur with the same efficiency as the forward process (a) of Figure 5(a). The time-reversed (or "back-action") process (b) is important because it allows the *generation* of gravitational radiation, and can therefore become a practical *source* of such radiation.

Hertz-like experiment

This raises the possibility of performing a Hertz-like experiment, in which process (b) becomes the source, and its reciprocal process (a) becomes the receiver, of gravity waves, as indicated in Figure 6.

Room-temperature Faraday cages, indicated by rectangles in Figure 6, prevent the transmission of electromagnetic waves, so that only gravitational waves, which can easily pass through all classical matter such as the normal (i.e., dissipative) metals of which standard, room-temperature Faraday cages are composed, are transmitted between the two halves of the apparatus that serve as the source and the receiver, respectively. Such an experiment would be practical to perform using standard microwave sources and receivers, if the scattering cross-sections and the transducer conversion efficiencies of the two charged quantum fluids are not too small.

An experiment using YBCO, which is a superconductor at liquid nitrogen temperatures, as the material for the two charged quantum-fluid transducers in

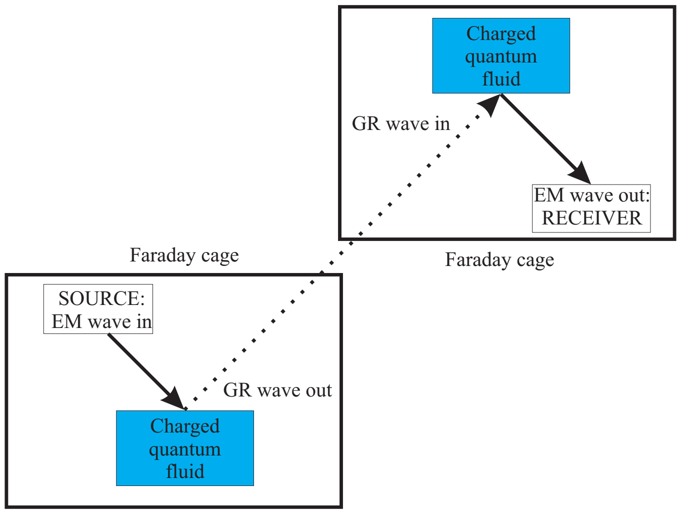

Figure 6: A Hertz-like experiment, in which a quantum transducer converts electromagnetic (EM) waves into gravity (GR) waves, and a second quantum transducer in the far field of the first back-converts gravity (GR) waves into detectable electromagnetic (EM) waves.

the Hertz-like experiment, has been performed at 12 GHz [6]. The conversion efficiency of each YBCO transducer in the two-transducer system, assuming that the two transducers are identical, has been measured to be less than 15 parts per million (probably due to the high microwave losses of YBCO, as compared to the extremely low characteristic impedance of free space for gravity waves, $Z_G = 16\pi G/c = 1.1 \times 10^{-17}$ SI units [6]).

Mössbauer-like response of "Millikan oil drops" in a magnetic trap to radiation fields

As a more practical realization of a quantum transducer using a charged quantum fluid, let us consider a pair of levitated "Millikan oil drops" in a magnetic trap, where the drops are separated by a distance on the order of a microwave wavelength, which is chosen so as to satisfy the impedance-matching condition for a good quadrupolar microwave antenna. See Figure 7.

Now let a beam of electromagnetic waves in the Hermite-Gaussian TEM_{11} mode [15], which has a quadrupolar transverse field pattern that has a substantial overlap with that of a gravitational plane wave, impinge at a 45° angle with respect to the line joining these two charged objects, as indicated in Figure 6. As a result of being thus irradiated, the pair of "Millikan oil drops" will be driven into motion in an anti-phased manner, so that the distance between them will

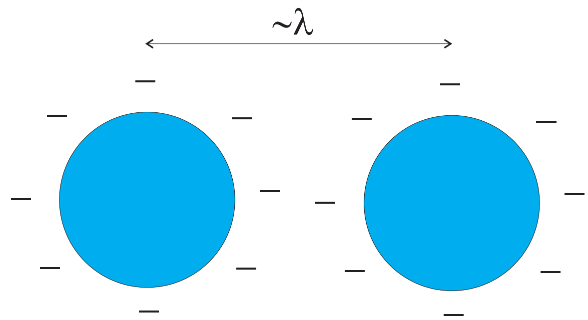

Figure 7: Two levitated "Millikan oil drops" in a magnetic trap.

oscillate sinusoidally with time, according to an observer at infinity [16]. Thus the simple harmonic motion of the two drops relative to one another produces a time-varying mass quadrupole moment at the same frequency as that of the driving electromagnetic wave. This oscillatory motion will in turn scatter (in a linear scattering process) the incident electromagnetic wave into gravitational and electromagnetic scattering channels with comparable powers, provided that the ratio of quadrupolar Larmor radiation powers given by Equation (15) is of the order of unity, which will be case when the mass of both drops is on the order of the critical mass m_{crit} for the case of single electrons attached to each drop. The reciprocal process should also have a power ratio of the order of unity.

We will now discuss the Mössbauer-like response of "Millikan oil drops." Imagine what would happen if one were replace an electron in the vacuum with a single electron which is firmly attached to the surface of a drop of superfluid helium (^{4}He) in the presence of a strong magnetic field and at ultralow temperatures, so that the system of the electron and the superfluid, considered as a single quantum entity, would form a macroscopic quantum ground state. Such a quantum system can possess a sizeable gravitational mass. For the case of many electrons attached to a massive drop, where a quantum Hall fluid forms on the surface of the drop in the presence of a strong magnetic field, there results a nondegenerate, Laughlin-like ground state.

In the presence of Tesla-scale magnetic fields, an electron is effectively prevented from moving at right angles to the local magnetic field line around which it is executing tight cyclotron orbits. The result is that the surface of the drop, to which the electron is tightly bound, cannot undergo liquid-drop deformations, such as the oscillations between the prolate and oblate spheroidal configurations of the drop which would occur at low frequencies in the absence of the magnetic field. After the drop has been placed into Tesla-scale magnetic fields at milli-Kelvin operating temperatures, both the single- and many-electron drop systems will be effectively frozen into the ground state, since the characteristic energy scale for electron cyclotron motion in Tesla-scale fields is on the order of

Kelvins. Due to the tight coupling of the electron(s) to the surface of the drop, this would freeze out all shape deformations of the superfluid drop.

Since all internal degrees of freedom of the drop, such as its microwave phonon excitations, will also be frozen out at sufficiently low temperatures, the charge and the entire mass of the "Millikan oil drop" should co-move rigidly together as a single unit, in a Mössbauer-like response to applied radiation fields. This is a result of the elimination of all internal degrees of freedom by the Boltzmann factor at sufficiently low temperatures, so that the system stays in its ground state, and only the external degrees of freedom of the drop, consisting only of its center-of-mass motions, remain.

The criterion for this "zero-phonon," or Mössbauer-like, mode of response of the electron-drop system is that the temperature of the system is sufficiently low, so that the probability for the entire system to remain in its nondegenerate ground state without even a single quantum of excitation of any of its internal degrees of freedom being excited, is very high, i.e.,

$$\text{Prob. of zero internal excitation} \approx 1 - \exp\left(-\frac{E_{\text{gap}}}{k_B T}\right) \to 1 \text{ as } \frac{k_B T}{E_{\text{gap}}} \to 0, \quad (17)$$

where E_{gap} is the energy gap separating the nondegenerate ground state from the lowest permissible excited states, k_B is Boltzmann's constant, and T is the temperature of the system. Then the quantum adiabatic theorem ensures that the system will stay adiabatically in the nondegenerate ground state of this quantum many-body system during perturbations, such as those due to weak, externally applied radiation fields. By the principle of momentum conservation, since there are no internal excitations to take up the radiative momentum, the center of mass of the entire system must undergo recoil in the emission and absorption of radiation. Thus the mass involved in the response to radiation fields is the mass of the whole system.

For the case of a single electron (or many electrons in the case of the quantum Hall fluid) in a strong magnetic field, the typical energy gap is given by

$$E_{\text{gap}} = \hbar\omega_{\text{cyclotron}} = \frac{\hbar e B}{mc} >> k_B T \ , \quad (18)$$

an inequality which is valid for the Tesla-scale fields and milli-Kelvin temperatures being considered here.

Estimate of the scattering cross-section

Let $d\sigma_{a\to\beta}$ be the differential cross-section for the scattering of a mode a of radiation of an incident gravitational wave to a mode β of a scattered electromagnetic wave by a pair of "Millikan oil drops." (We shall denote GR waves by Roman-letter subscripts, and EM waves by Greek-letter subscripts.) Then, by time-reversal symmetry

$$d\sigma_{a\to\beta} = d\sigma_{\beta\to a} \ . \quad (19)$$

Since electromagnetic and weak gravitational fields both formally obey Maxwell's equations [17] (apart from a difference in the signs of the source density and the source current density), and since these fields obey the same boundary conditions, the solutions for the modes for the two kinds of scattered radiation fields must also have the same mathematical form. Let a and α be a pair of corresponding solutions, and b and β be a different pair of corresponding solutions to Maxwell's equations for GR and EM modes, respectively. For example, a and α could represent incoming plane waves which copropagate in the same direction, and b and β scattered, outgoing plane waves which copropagate together in a different direction. Then for the case of a pair of critical-mass drops with single-electron attachment, there is an equal conversion into the two types of scattered radiation fields in accordance with Equation (15), and therefore

$$d\sigma_{a\to b} = d\sigma_{a\to\beta} \ , \tag{20}$$

where b and β are corresponding modes of the two kinds of scattered radiations.

By the same line of reasoning, for this pair of critical-mass drops

$$d\sigma_{b\to a} = d\sigma_{\beta\to a} = d\sigma_{\beta\to\alpha} \ . \tag{21}$$

It therefore follows from the principle of reciprocity (i.e. time-reversal symmetry) that

$$d\sigma_{a\to b} = d\sigma_{\alpha\to\beta}. \tag{22}$$

In order to estimate the size of the total cross-section, it is easier to consider first the case of electromagnetic scattering, such as the scattering of microwaves from two critical-mass drops, with radii R and a separation r on the order of a microwave wavelength. Let the electrons on the "Millikan oil drops" be in a quantum Hall plateau state, which we know is that of a perfectly dissipationless quantum fluid, like that of a superconductor. Furthermore, we know that the nondegenerate Laughlin ground state is that of a perfectly rigid, incompressible quantum fluid [18]. The two drops thus behave like perfectly conducting, shiny, mirrorlike spheres, which scatter light in a manner similar to that of perfectly elastic hard-sphere scattering in idealized billiards. The total cross section for the scattering of electromagnetic radiation from a pair of drops is therefore given approximately by the geometric cross-sectional areas of two hard spheres

$$\sigma_{\alpha\to\text{all }\beta} = \int d\sigma_{\alpha\to\beta} \simeq \text{Order of } 2\pi R^2 \tag{23}$$

where R is the hard-sphere radius of a drop.

However, if, as one might expect on the basis of classical intuitions, that the total cross-section $\sigma_{a\to\text{all }b}$ for the scattering of gravitational waves from the two-drop system would be extremely small, like that of all classical matter such as the Weber bar, then by reciprocity, the total cross-section $\sigma_{\alpha\to\text{all }\beta}$ for the scattering of electromagnetic waves from the two-drop system must also be extremely small. This would lead to a contradiction with the hard-sphere cross section given by Equation (23), so these intuitions must be incorrect.

From the reciprocity principle and from the important properties of quantum rigidity and quantum dissipationlessness of these drops, one therefore concludes that for two critical-mass "Millikan oil drops," it must be the case that

$$\sigma_{a \to \text{all } b} = \sigma_{\alpha \to \text{all } \beta} \simeq \text{Order of } 2\pi R^2 \ . \tag{24}$$

Signal-to-noise considerations

The signal-to-noise ratio expected for the Hertz-like experiment depends on the current status of microwave source and receiver technologies. Based on the experience gained from the experiment done on YBCO using existing off-the-shelf microwave components [6], we expect that we would need geometric-sized cross-sections and a minimum conversion efficiency on the order of a few parts per million per transducer, in order to detect a signal.

It should be stressed that in the Hertz-like experiment, one is not trying to detect the *strain* of space (which may be extremely small), but rather the *power* that is being transferred by radiation from one quantum transducer to the other. The overall signal-to-noise ratio depends on the initial microwave power, the scattering cross-section, the conversion efficiency of the quantum transducers, and the noise temperature of the microwave receiver (i.e., its first-stage amplifier).

Microwave low-noise amplifiers can possess noise temperatures that are comparable to room temperature (or even better, such as in the case of liquid-helium cooled paramps used in radio astronomy). The minimum power P_{min} detectable in an integration time τ is given by

$$P_{\text{min}} = \frac{k_B T_{\text{noise}} \Delta\nu}{\sqrt{\tau \Delta\nu}} \tag{25}$$

where k_B is Boltzmann's constant, T_{noise} is the noise temperature of the first stage microwave amplifier, and $\Delta\nu$ is its bandwidth. Assuming an integration time of one second, and a bandwidth of 1 GHz, and a noise temperature $T_{\text{noise}} =$ 300 K, one gets $P_{\text{min}}(\tau = 1 \text{ sec}) = 1.3 \times 10^{-25}$ Watts.

Why such an enormous enhancement?

Why is there such an enormous enhancement of over 40 orders of magnitude in the quantum transducer conversion efficiency predicted by Equation (15) for two "Millkan oil drops" over that for two electrons in the vacuum separated by the same distance, predicted by Equation (10)?

The answer is that the macroscopic quantum phase coherence of superfluid helium allows an enormous number of atoms in the superfluid to all move together coherently in unison in response to gravitational radiation, so that there exists an enormous enhancement of the oscillating mass quadrupole moment of

the two drops by a factor of N_{atom}, the number of atoms participating in the time-varying superposition of the center-of-mass momentum eigenstates of these drops induced by the radiation. There is a corresponding enhancement in the amount of gravitational radiation power that is emitted by a pair of "Millikan oil drops" over that emitted by a pair of bare electrons separated by the same distance in the vacuum, by a factor of N_{atom}^2. In the case of the Planck mass, $N_{atom} \sim 10^{18}$ helium atoms, and in the case of the critical mass, $N_{atom} \sim 10^{17}$ helium atoms. At a fundamental level, this enormous enhancement originates from the superposition principle of quantum mechanics.

Here I am assuming that there does not exist any appreciable decoherence of quantum superposition states which contain a sufficiently large amount of gravitational mass so that the superposition principle of quantum mechanics breaks down. It has been suggested that stochastic backgrounds of gravity waves from the Big Bang acting on quantum superpositions at the Planck mass scale may indeed cause such a decoherence [19]. The Hertz-like experiment, if properly performed, may be a test of the validity of the superposition principle of quantum mechanics for Planck-mass objects such as "Millikan oil drops." I hope to be able to perform the Hertz-like experiment with my colleagues at Merced.

Acknowledgments

My appreciation goes to Marlan Scully for awarding me the Willis Lamb medal. I would also like to thank Gene Commins, Marc Feldman, Giorgio Frossati, John Garrison, Dan Gauthier, Øyvind Grøn, Dave Kelley, Wolfgang Ketterle, Manfred Kleber, Jon Magne Leinaas, Robert Littlejohn, Peter Milonni, Kevin Mitchell, Richard Packard, Bill Phillips, Paul Richards, Ray Sachs, Wolfgang Schleich, Achilles Speliotopoulos, Cliff Taubes, Peter Toennies, Dave Wineland, and Roland Winston for helpful discussions. I would also like to thank all my former students, especially Ivan Deutsch, Paul Kwiat, and Aephraim Steinberg, for organizing the recent "Chiaofest," and to thank all my students, collaborators and colleagues, especially Charles Townes, for honoring me by their presence at this celebration in Snowbird.

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$$-\frac{dE}{dt} = \frac{G}{45c^5}\dddot{D}_{ij}^2 \tag{26}$$

where E is the energy of the orbiting neutral object, the triple dots denote the third derivative with respect to time of the mass quadrupole-moment tensor D_{ij}. One finds that

$$\kappa = \frac{2}{15}\frac{v^2}{c^2} \tag{27}$$

where v is the orbital velocity of the neutral object. Since $v << c$ for the orbital motion of the neutral object around the Earth, $\kappa << 1$.

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$$\mathbf{E}(\mathbf{r}, t) = \lim_{q \to 0} \frac{\mathbf{F}(\mathbf{r}, t)}{q} \tag{28}$$

where $\mathbf{F}(\mathbf{r}, t)$ is the force acting on test charge q located at $\mathbf{r}$ at time t; see W. K. H. Panofsky and M. Phillips, *Classical Electricity and Magnetism* (Addison-Wesley, Reading, MA, 1955), Equation (1-23) and its following discussion. By "vanishingly small," we mean here that one can neglect the radiation-reaction force arising from any kind of radiation (either electromagnetic or gravitational) emitted by the object q.

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approximately, but not truly exactly, freely falling drops would radiate gravitational radiation, in agreement with the observations of the observer at infinity. The drops can therefore have very large masses, and hence very large amounts of inertia, and yet still emit gravity waves (assuming that they have a fixed charge-to-mass ratio so that Equation (16) is satisfied, and that gravitational nonlinearities are negligible).

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